Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: Topics from §1.3-§1.5

Discussions 201, 203 // 2018-09-04

1. New functions from old

Problem 1. Visually, if you start with the graph of $y = \sin(x)$ and shift it to the left by $\pi/2$, you get the graph of $y = \cos(x)$. Express this relationship algebraically.

Problem 2. Let $f(x) = x^2 + 1$, $g(x) = \tan(x)$, and $h(x) = x - \pi/2$.

(1) Compute $(h \circ f)(\pi)$ and $(f \circ h)(\pi)$. Are they the same?
(2) Compute $(f \circ f \circ f \circ f)(0)$.
(3) Find $f \circ g \circ h$ and simplify as much as possible.

Problem 3. Suppose the function f has domain $[0, 3]$ and range $[3, 7]$ while the function g has domain $[3, 7]$ and range $[1, 2]$. Can you determine the domain and range of $g \circ f$, or is there not enough information? How about $f \circ g$?

What if (keeping everything else the same) you were instead told that the range of g is $[0, 4]$?

2. Exponential functions

Problem 4. Compute $2^{(2^2)}$ and $\left(2^2\right)^2$. Are they the same?

Problem 5. Answer the same question for $3^{(3^3)}$ and $\left(3^3\right)^3$. Although this time you may want to use a calculator—one of these expressions is quite large.

Remark. By convention, an expression of the form a^{b^c} without any parentheses means $a^{(b^c)}$.

Problem 6. Let a and b be positive constants, neither of which is equal to 1. Describe precisely what visual transformation(s) must be applied to the graph of $y = a^x$ to obtain the graph of $y = b^x$. (Do you stretch? squeeze? shift? reflect? and in what direction? by what factor or amount?)

Problem 7. In a laboratory experiment, a radioactive sample is allowed to decay. The amount left after a duration t has elapsed is given by the equation $f(t) = Ab^t$. At time $t = 2$, there are 100 units of the sample left. At time $t = 3$, there are only 25 units. What are A and b?

Now, instead of writing $f(t) = Ab^t$, express the same function in the form

$$f(t) = C \cdot \left(\frac{1}{2}\right)^{t/\lambda}.$$

What are C and λ, and what is their physical significance?

3. Inverse functions

Problem 8. For each of the following functions, determine its domain, range, and whether it is one-to-one. If it is indeed one-to-one, compute its inverse. Graph the function, and its inverse if applicable.

(1) $f(x) = x^3 + 2$
(2) $f(x) = 5/x$
(3) $f(x) = 5/x^2$
(4) $f(x) = x^2 - 2x + 13$ (Hint: to compute the range, try rewriting the function as $f(x) = a(x - b)^2 + c$. First find a, then find b, and finally find c. This technique is known to some as "completing the square.")

Problem 9. Compute $\sin(\tan^{-1}(12/5))$.

Problem 10. Evaluate the product

$$(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 6)\cdots(\log_{30} 31)(\log_{31} 32).$$